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Weighing Positive vs. Negative Factors: A Data Envelopment Analysis Technique

Decision making

When deciding on actions to undertake, we face a problem:

Are the outcomes – actual or expected – worth the efforts that should be undertaken?

Conventionally, we use subtraction or division to support our decision quantitatively.

Examples:

Should we start a business?

Revenue – Expenses = Profit (Yes if +, No if -)

Should we make an investment?

Interest / Investment *100 = Interest Rate

(Yes, if greater than S&P + 4%, otherwise, no)

Case of several factors

If several factors come into play, for example, when we need to choose the best player or a team, then the problem becomes more complicated, and cannot be resolved in one step.

Operations Research (OR) suggests several ways of solution to such problems. Data Envelopment Analysis (DEA) is one of the most popular techniques, Charnes et al. (1978), Banker et al. (1984), Cooper et al. (2011).

DEA uses a ratio of weighted sums of outputs (positive factors) and inputs (negative factors) and finds the weights providing maximum value of the ratio.

$$E = \frac{\sum_{i=1}^{r} u_i Y_i}{\sum_{j=1}^{s} v_j X_j}$$

where *E* is an efficiency score , Y_i are outputs, X_j – inputs, and u_i and v_j are weight coefficients that DEA determines.

Educational concern

DEA uses Linear Programming (LP) theory and algorithms to determine the coefficients. Typical Quantitative Reasoning students do not study this mathematical tool.

Publications Vaninsky (2009, 2011 a,b) suggest a way to avoid LP.

They suggest constructing a virtual Perfect Object (PO) that has maximum observed outputs and minimum observed inputs and use it as a benchmark for comparison of the actual objects.

The derived formula is:

E = (Maximum relative output)/(Minimum relative input),

where "relative " means "expressed in terms of Perfect Object"

The formula is very simple, and requires merely a scientific calculator.

Example.

Finding a team winner of the XXX Olympic Games in London, 2012

Statistics.

A total of 10,961 athletes representing 205 countries participated in the Games. They were awarded with 958 medals: 304 gold, 297 silver, and 357 bronze. The averages are: 4.67 medals per country and 11.442 athletes per medal; only 87 countries won at least one medal.

Statistical data is available at the web sites

http://www.olympic-2012-london.co.uk/london-olympycs-2012-countries-statistics.php

http://en.wikipedia.org/wiki/List_of_countries_by_population

To find a team winner, we put the number of total and gold medals against a team size and population of a country.

Example of calculations

Step 1. Collect information and create a Perfect object

Country	Gold medals Output-1	Total medals Output-2	Team size Input-1	Population, mln Input-2
(1)	(2)	(3)	(4)	(5)
USA	*46*	*104*	539	314.147
Chi	38	87	385	1347.350
Jam	4	12	53	*2.706*
Eth	3	7	*35*	84.321
PO	46	104	35	2.706

Step 2. Find relative outputs and inputs, take max output and min input, and calculate the efficiency score as their ratio

Country	Relative Output-1	Relative Output-2	Max Relative Output	Relative Input-1	Relative Input-2	Min Relative Input	Efficiency	Rank
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
USA	1.000	1.000	*1.000*	15.400	116.093	*15.400*	0.065	4
Chi	0.826	0.837	0.837	11.000	497.912	11.000	0.076	2
Jam	0.087	0.115	0.115	1.514	1.000	1.000	0.115	1
Eth	0.065	0.067	0.067	1.000	31.161	1.000	0.067	3

Calculations for the USA:

Relative outputs 46/46=1.000; 104/104=1.000; *max*(1.000, 1.000) = 1.000

Relative inputs 539/35 = 15.400; 314.147/2.706 = 116.093; *min* (116.093, 15.400) = 15.400

Efficiency = 1.000/15.400 = 0.065

Actual data

We restricted our considerations to the countries that won at least 4 total and 2 gold medals. The following efficiencies and ranks were obtained for the 10 best teams:

Country	Efficiency	Efficiency rank
(1)	(2)	(3)
Jamaica	0.1154	1
Iran	0.0808	2
New Zealand	0.0763	3
China	0.0760	4
Kenya	0.0740	5
Ethiopia	0.0673	6
USA	0.0649	7
Russia	0.0626	8
Romania	0.0571	9
Hungary	0.0472	10

Discussion

- **The Jamaican team was very successful in track and field, and won 12 Olympic medals with 4 gold. Jamaican sprinters became Olympic stars. The team won an Olympic medal per 4.41 team members, while the average number is one medal per 11.4 team members. The ratio of population (2.706 million) to the team size is just 51.1, while it is 433.5 for the sample.**

- **Iran is very close to Jamaica by medals and team size: 4 gold, 12 total, 50 people team, but its population much greater: 75.15 million.**

- **As Iran and Jamaica are quite different countries, one can speculate that a team of 50 - 53 is optimum for the countries with limited Olympic budget.**

- **Efficiencies of China (385-member team, ranked 4^{th}), United States (539, 7^{th}), Russia (441, 8^{th}), and Great Britain (558, 17^{th}) are much lower. These countries experienced diminishing returns to scale. This notion deserves a special discussion in more general settings.**

Possible applications of the suggested approach

- **Environmental efficiency with gross domestic product (GDP) and population as outputs and energy consumption and CO2 emissions as inputs;**

- **Firm evaluation with profits and sales as outputs and equity and debt as inputs;**

- **Colleges comparison with number of students and graduation rate as outputs, and number of faculty, staff, and operational expenses as inputs;**

- **Car evaluation with miles per gallon and safety score as outputs, and price and maintenance expenses as inputs.**

Educational features of DEA PO

When studying DEA, students learn to separate positive factors (outputs) from negative factors (inputs) and to weigh them against each other.

DEA PO processes objects independently, one at a time. This allows for the subdivision of the task among the students working independently and combining their results for analysis and discussion after the work has been done.

Simplicity of calculations allow using DEA PO practically at any level of education with the stress on decision making.

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Thank you !!!

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