Don't chloroform the child's reasoning faculties: A little-known mathematics curriculum based on meaning

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The number system has little meaning for students

Estimate 3.04×5.3

1.6
16
160
1600

The number system has little meaning for students

Estimate 3.04×5.3

	Age 13
1.6	28%
16	21
160	18
1600	23
No answer	9

The number system has little meaning for students

Estimate 3.04×5.3

	Age 13	*Age 17*
1.6	28%	21%
16	21	37
160	18	17
1600	23	11
No answer	9	12

We can solve the most fundamental problem in education: rote learning

We can solve the most fundamental problem in education: rote learning

1. Rote learning is a plague upon education.
2. Benezet showed us how to solve this fundamental problem.
3. His meaning-based curriculum was based on readiness, estimation, and comparison.

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Rote learning is the victory of symbol manipulation over meaning

$$\frac{243 + 243 + 243}{3} = ?$$

Rote learning is the victory of symbol manipulation over meaning

$$\frac{243 + 243 + 243}{3}$$

laughter too hard!

$$\begin{array}{r} 243 \\ 243 \\ +\,243 \\ \hline 729 \end{array}$$

Rote!

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Benezet recognized the prevalence of rote learning

'For some years I had noted that the effect of the early introduction of arithmetic had been to dull and almost chloroform the child's reasoning faculties.'

Louis P. Benezet
Superintendent of schools
Manchester, New Hampshire

Benezet abolished formal arithmetic teaching until seventh grade

Benezet's curriculum faced a class barrier

'[M]ore carefully than I picked the teachers, I selected the schools.

Three of the four schoolhouses involved were located in districts where not one parent in ten spoke English as his mother tongue. I sent home a notice to the parents and told them about the experiment that we were going to try, and asked any of them who objected to it to speak to me about it.

I had no protests. Of course, I was fairly sure of this when I sent the notice out. Had I gone into other schools in the city where the parents were high school and college graduates, I would have had a storm of protest and the experiment would never have been tried.'

Instead of algorithms, students learned thinking

The children were no longer under the restraint of learning multiplication tables or struggling with long division.

[Instead they] were encouraged to do a great deal of oral composition. They reported on books that they had read, on incidents which they had seen, on visits that they had made. They told the stories of movies that they had attended and they made up romances on the spur of the moment.

Mathematics was simply a branch of thinking and meaning making

1. Numbers < 100; comparison: more, less, higher, lower.
2. Telling time (hours and half-hours); page numbers; using an index.
3. Bigger numbers: license plates, house numbers.
4. Inch, foot, yard. Estimating lengths. Square inch, square foot.
5. Counting by 5s, 10s, 2s, 4s, and 3s (mentally), leading to those multiplication tables. Estimation games; always writing estimate before checking. Fractions by pictures. Multiplication table.
6. Formal arithmetic but estimate first then check.
7. Lots of mental arithmetic without reference to paper or blackboard.
8. More mental arithmetic. Reasons for processes. Explaining how to attack problems.

Quantitative assessments supported the change

'The results of this study cast doubt upon whether we are justified in devoting five years to the drilling of formal arithmetic.'

Berman, Etta (1935). *The Result of Deferring Systematic Teaching of Arithmetic to Grade Six as Disclosed by the Deferred Formal Arithmetic Plan at Manchester, New Hampshire*. Masters Thesis, Boston University, USA.

Quantitative assessments supported the change

The distance from Boston to Portland by water is 120 miles. Three steamers leave Boston, simultaneously, for Portland. One makes the trip in 10 hours, one in 12, and one in 15. How long will it be before all 3 reach Portland?

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Classroom	*Grade*	*Score*
Experimental	2	nearly perfect
Traditional	9	only 6 correct out of 29

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'In one [traditionally taught] fourth grade I could not find a single child who would admit that he had committed the sin of reading. I did not have a single volunteer, and when I tried to draft them, the children stood up, shook their heads, and sat down again.

In the four experimental fourth grades the children fairly fought for a chance to tell me what they had been reading. The hour closed, in each case, with a dozen hands waving in the air and little faces crestfallen, because we had not gotten around to hear what they had to tell.'

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Benezet's curriculum is designed around two crucial principles

The first design principle is the zone of proximal development

'If some of the children do not grasp the problems easily and quickly, the teacher simply passes on, knowing that the power to reason will probably develop in them a year or two subsequently.

The one thing which is avoided is that children shall get the idea that a fixed method or formula can be used as a substitute for thinking.'

Waiting till children are ready mitigates rote learning

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The second design principle is estimation and comparison

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Estimation undercuts rote learning

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Comparison implies ratio

Is area A more than twice area B? $\equiv \frac{A}{B} > 2.$

Ratios match the structure of the world

A ratio (of lengths, areas, prices, . . .) is **dimensionless**.

When our system of units changes, a ratio does not change.

Therefore, the universe cares only about dimensionless quantities.

Comparison and ratio match our neurobiological hardware

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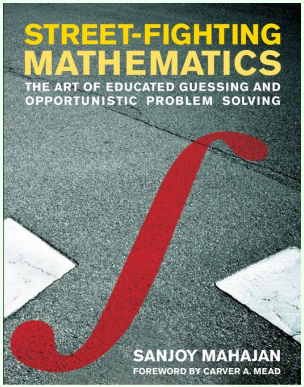
In designing our curricula, we can learn much from Benezet.

Several resources are freely available

Benezet's curriculum

`www.inference.phy.cam.ac.uk/sanjoy/benezet/`
or Internet search for *"Benezet curriculum"*



On dimensions and estimation
Sections 1.1, 2.4, 3.1, 5.1
`streetfightingmath.com`

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Benezet's experiment lasted almost 15 years

1924: 5-year appointment

1929: 6-year reappointment

1933: Motion to kill program fails

1935: 3-year reappointment

1935: Motion to follow textbooks passes

1938: Benezet leaves for Dartmouth