Ten years from now our students should . . .

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Sample problems from *Common Sense Mathematics* (`http://www.cs.umb.edu/~eb/qrbook/qrbook.pdf`. Course home page: `http://www.cs.umb.edu/~eb/114/`.

Exercise 1. On May 4, 2010 Olivia Judson wrote in *The New York Times* that

> [Baba] Brinkman, a burly Canadian from Vancouver, is a latter-day wandering minstrel, a self-styled "rap troubadour," with a master's degree in English and a history of tree-planting (according to his Web site, he has personally planted more than one million trees). [2]

How long would it take to personally plant a million trees? Is Brinkman's claim reasonable?

Exercise 2. A bumper sticker available in 2007 at `http://donnellycolt.com` claims that

> Every Minute the World Spends $700,000 on War While 30 children Die of Hunger & Inadequate Health Care

Are the figures $700,000 and 30 children believable?

Exercise 3. Paying for Harvard

Several years ago *The Boston Globe* reported on a man who put his son through four years at Harvard University by collecting and redeeming soda cans.

Do you believe this is possible?

Exercise 4. Scrap the penny?

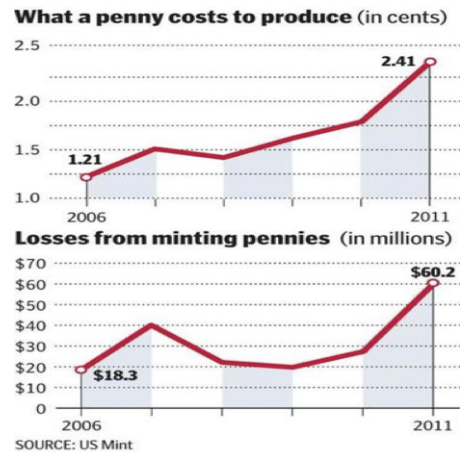*The Boston Globe* reported on April 19, 2012, that

> [MIT physicist Jeff] Gore, who has become an unlikely national spokesman for efforts to eliminate the penny, is gaining fresh hope that the United States might finally dump the coin. Canada recently announced it would get rid of its one-cent piece by the end of the year even as the US Mint reported it is losing more money than ever to produce the coin. [3]

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[2] `http://opinionator.blogs.nytimes.com/2010/05/04/darwin-got-it-going-on/`

[3] `http://www.bostonglobe.com/business/2012/04/18/mit-physicist-jeff-gore-leads-effort-scrappenny/Xz04E54R4BTkoDTYIOpnJO/story.html`

These graphs accompanied the article:

What a penny costs to produce (in cents)

2.5
2.41
2.0
1.5
1.21
1.0
2006   2011

Losses from minting pennies (in millions)

$70
$60.2
$60
$50
$40
$30
$20
$18.3
$10
0
2006   2011

SOURCE: US Mint

(a) Use data you find in the graphs to estimate the number of pennies minted in 2011.

(b) The print version of the article says that 4,829 million pennies shipped in 2011. Is that answer consistent with your estimate?

(c) How can you tell from the graphs *without doing any arithmetic* that there were fewer pennies minted in 2009 than in 2008?

Exercise 5. How long is a microcentury?

The mathematician John von Neumann is often identified as the source of the fact that a 50 minute lecture is about a microcentury.

(a) Check this fact by doing your own arithmetic.

(b) What are the absolute and relative errors in the "about"?

(c) If you could listen to lectures one after another day in day out for a century about how many would you hear?

(d) Estimate how many years it would take for professors at your school to have delivered a century's worth of classes. (Don't assume the classes were back to back year 'round.)

Exercise 6. Gaming the system

In the *Chess Notes* column in *The Boston Globe* on Tuesday, March 10, 2008 Harold Dondis and Patrick Wolff wrote that

... the sensation of the [Amateur East] tourney was the team with the highest score, GGGg (no relation to the song Gigi but standing for the three Grandmasters and one future Grandmaster.) The players were our Eugene Perelshteyn, Roman Dzindzichasvili, Zviad Izoria, all Grandmasters, and 5-year-old Stephen Fanning who rounded out the team. Was this a valid lineup? Well, yes, it was. The rules of the Amateur provide that the average rating of a team could not exceed 2200. GGGg's three Grandmasters were well above that, but Stephen Fanning ... had a current rating of 178, which brought the average rating to 2017.

The Grandmasters of GGGg ... delivered wins, while their fourth board, who would have won the prize as the cutest chess player (if the sponsors had had the foresight to establish such a prize) struggled to make legal moves, sometimes failing to do so. Naturally, there followed an extensive debate as to whether the victorious ensemble had gamed the system.

(a) If the three Grandmasters had the same rating, what would it have been?

(b) Did GGGg game the system?

(c) How might the tourney organizers change the rules to prevent this kind of team from winning?

Exercise 7. Household income in the United States

The histogram in the following figure [4] shows the percentage of the population in income groups $10,000 increments apart, except for the furthest two right columns which correspond to increments of $50,000. The data are from 2005.
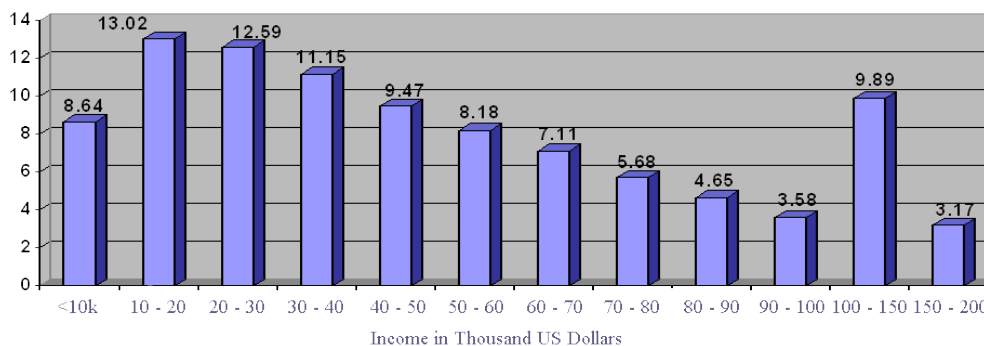

(a) Enter the data in an Excel spreadsheet.

(b) Build a histogram in Excel that comes as close as possible to matching the one in wikipedia. Display it two ways, with and without the phony three dimensional effect.

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[4] From http://en.wikipedia.org/wiki/Household_income_in_the_United_States)

(c) Do the percentages sum to 100%? If not, what might explain the discrepancy?

(d) Estimate mean, median and modal household income. Then discuss how your answers compare to the corresponding figures in wikipedia.

You can find the raw data at the US Census 2006 Economic Survey (`http://pubdb3.census.gov/macro/032007/hhinc/new06_000.htm`) but you should not need it to work this exercise.

Exercise 8. When you apply the brakes in a Toyota Prius the car uses some of the energy of the forward motion to recharge the battery. The figure shows how the dashboard displays a little car icon each time that recharging has collected 50 watt-hours. The figure shows nine of those icons. [5]
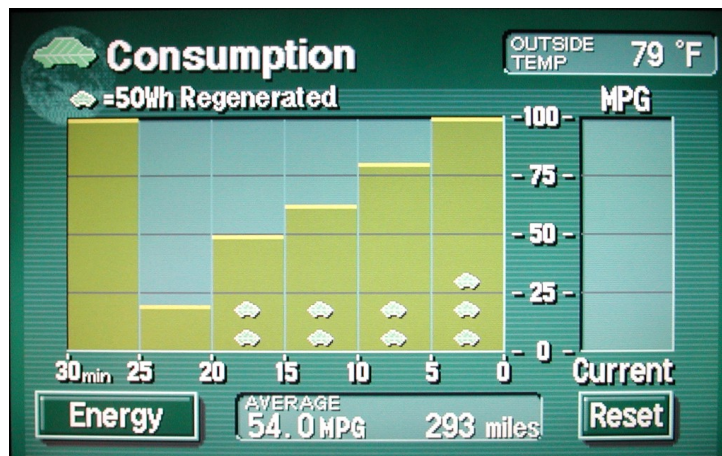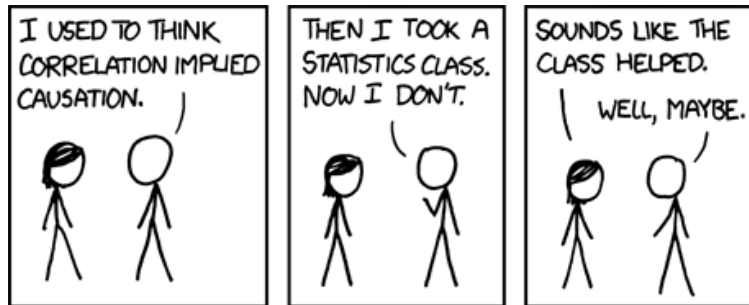

(a) Estimate the energy equivalent of each icon in gallons of gasoline.

(b) Estimate the dollar value of that gasoline.

(c) Compare your estimate to the dollar value of 50 watt-hours of electricity in your house (what it would cost to keep a 100 watt bulb on for half an hour).

(d) Discuss the value of the display.

Exercise 9. Well, maybe.

Explain the joke in this cartoon from `http://xkcd.com/`.

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[5]The image is from `http://nudges.wordpress.com/2008/05/11/does-the-prius-fuel-gauge-change-driving-habits` with permission. The text there is interesting too.

Exercise 10. *Educating mothers saves lives, study says*

From *The Boston Globe*, September 17, 2010

> By using statistical models, the researchers found that for every extra year of education women had, the death rate for children under 5 dropped by almost 10 percent. In 2009, they estimated that 4.2 million fewer children died because women of childbearing age in developing countries were more educated.
>
> In 1970, women aged 18 to 44 in developing countries went to school for about two years. That rose to seven years in 2009. [6]

(a) How much did the death rate for children under 5 decline from 1970 to 2009?

(b) Build as much as you can of the exponential model implicit in this quotation. What are the independent and dependent variables? What is the annual relative change?

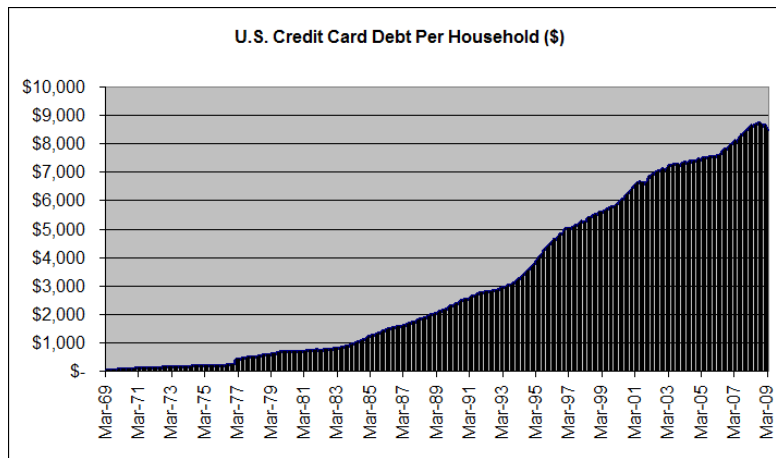**Exercise 11.** Average household credit card debt.

On May 5, 2011 the Boston Metro (a free daily paper available at subway stops) carried an advertorial that read (in part)

> Every three minutes another person falls behind on credit card debt. ... [T]he growing mountain of consumer debt [now] stands at $2.17 trillion.

along with the graph below[7].

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[6]http://www.boston.com/news/world/europe/articles/2010/09/17/educating_mothers_saves_lives_study_says/

[7]That graph comes from http://chartingtheeconomy.com/?p=1174=1 where the U.S. Federal Reserve is indicated as the data source. The actual Metro article used a redrawing of the same graph.

U.S. Credit Card Debt Per Household ($)

(a) What's an advertorial?

(b) Is the claim that "every three minutes another person falls behind ..." reasonable?

(c) Is the $2.17 trillion figure in the quotation consistent with the numbers in the graph?

(d) Which average household credit card debt is plotted on the vertical axis – the mean, the median or the mode? (Explain why you think so.)

(e) The data points since 1989 show approximately linear growth. Estimate the slope of the regression line (trendline), in dollars per household per year. (If you had the numbers in a spreadsheet you could ask Excel for that slope, but you can get a reasonable estimate from the picture.)

(f) Use your answer to the previous part of the problem to predict average household credit card debt for the years 2011 and 2019. How reliable do you think your predictions are?

Exercise 12. *Possible cut to beach testing a health threat, critics say*

In an article in *The Boston Globe* reprinted from the *Washington Post* on March 4, 2012 you could read that

> The average citizen visits a coastal shore, Great Lake, or river about 10 days a year, according to a federal estimate ...
> About 3.5 million people each year get sick enough to be nauseated or get diarrhea after splashing in water containing harmful bacteria, according to an Environmental Protection Agency estimate. [8]

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[8]http://www.bostonglobe.com/news/nation/2012/03/04/elimination-funding-for-beach-contamination-monito 9sGB4SzlU2m3CvM6jaINhN/story.html

What is the probability that a visit to the beach will make you sick?

Exercise 13. Misinformation on the web

From `http://www.numbersplanet.com/`, a discussion about Powerball drawings and statistics about odd numbers drawn compared to even numbers drawn.

> ```
>    4180 Odd numbers have been drawn, and 4178 Even numbers have been drawn.
> 6 Even / 0 Odd Drawings: 17
> 5 Even / 1 Odd Drawings: 127
> 4 Even / 2 Odd Drawings: 331
> 3 Even / 3 Odd Drawings: 431
> 2 Even / 4 Odd Drawings: 353
> 1 Even / 5 Odd Drawings: 118
> 0 Even / 6 Odd Drawings: 16
> ```
> The important statistic is the `3 Even / 3 Odd: 431` statistic ... [t]elling you that you should be playing six numbers that has a `3 Even and 3 Odd` number combination.

(a) Plot the data and show that it approximates the normal bell curve, as it should.

(b) Criticize the argument suggesting that you should always bet on a three even three odd combination.

(c) Fix the author's grammar.

Exercise 14. Breast cancer screening.

In his *Chances Are* blog in *The New York Times* on April 25, 2010 Steven Strogatz wrote about a diagnostic puzzle presented to several doctors:

> The probability that [a woman in this cohort] has breast cancer is 0.8 percent. If a woman has breast cancer, the probability is 90 percent that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7 percent that she will still have a positive mammogram. Imagine a woman who has a positive mammogram. What is the probability that she actually has breast cancer?
> ...
> [When 24 doctors were asked this question], their estimates whipsawed from 1 percent to 90 percent.   Eight of them thought the chances were 10 percent or less, 8 more said 90 percent, and the remaining 8 guessed somewhere between 50 and 80 percent. Imagine how upsetting it would be as a patient to hear such divergent opinions. [9]

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[9]`http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/`

(a) What is the correct answer?

  Hint: Build the contingency table, based on a population of 1,000 women tested.

(b) What percentage of the 24 doctors got the correct answer?